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Applications of Dimensionless Variables to Scaling in the Infrared

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ABSTRACT

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A particular scaling error can be expected to occur when modeling a long ship with a thin hull is presented. In such a case, if a scale of length is chosen that makes the model a convenient laboratory length, the same scale applied to the hull cross section may reduce it to one that is extremely thin, thus increasing costs and experimental inconvenience. A possible solution to such difficulties is presented based on using different scales for model hull length and thickness. The validity of employing two such scales of length depends upon the approximation that heat flux parallel to the hull may be neglected in comparison to that orthogonal to the hull.

The design of scaled models for the IR requires a choice of structural materials with suitable thermodynamic properties. Which thermodynamic properties are relevant are pointed out by examining the form of the controlling dimensionless variables. These include the thermal conductivity, mass density, specific heat, convective heat transfer coefficient, and emissivity, respectively symbolized in standard notation as k, ρ, c, h_c, and ϵ. Although the ranges of variation for these properties are readily available in standard texts and references, they are widely dispersed among the works and not listed in one consistent set of units. These properties are collected here in a convenient set of tables and one consistent set of units.

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INTRODUCTION

Scale models have long been of great utility in many branches of science and engineering. For example, the fields of hydrodynamics, aerodynamics, acoustics, and radar routinely employ scale model studies. Recently it has been recognized that scale models can be useful in the study of the IR contrast of ships. For this purpose, the laws of similitude have been derived that make possible the rational design of IR scale models of naval ships.* A brief review of these laws is presented in the next section, along with important concepts and equations.

Perfect geometrical similitude between a full-scale ship and its scaled model is one of the necessary requirements for the laws of IR scaling to hold. In particular, every linear dimension must be scaled by the same factor. Thus the length of a ship hull must be reduced by the same factor as the hull thickness.

In certain cases a scale model of convenient length corresponds to a model hull thickness too thin to be practical. Such a problem may be overcome through sufficient financial and experimental resources. For example, it might be possible to utilize unusual structural materials that would allow fabrication of a very thin hull, and still have sufficient strength to withstand the forces of a vigorous test program. Unfortunately, this would be both expensive and time consuming. Therefore it must be determined if it is possible to scale the hull length and hull thickness differently, and still find a sensible interpretation for the IR scaling experiment. This is a reasonable procedure, especially in the beginning stages of an IR experiment in which this problem may arise.

Initial rough experiments could be performed within the approximations required by simultaneous use of two different spatial scales. Then, if the experimental results proved beneficial, a subsequent larger expenditure of funds and effort to build a model that adhered exactly to geometrical scaling would be justifiable. In the meantime, an initial study in the IR could be pursued based upon a model that would be relatively inexpensive and quick to produce. Such a possibility is presented which is predicated upon the approximate validity of the assumption that the principal heat fluxes move in directions directly across the hull structure, i.e., not parallel but orthogonal to the surface.

In addition, the range of thermodynamic properties associated with common structural materials and conditions of interest for designing IR scaled experiments are surveyed. On examining the list of dimensional variables associated with the description of such experiments, it occurs that the relevant properties are k, ρ, c, α, h_c, and ϵ. Hence, the range of values expected for these variables are displayed in tables at the end of this report.

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* "Laws of Infrared Similitude," Peter O. Cervenka and Lou Massa, CDNSWC Report CARDIVNSWC-TR-94/002 (Jan 1994).
"Analysis of Infrared Scaling Laws," Lou Massa and Peter O. Cervenka, CDNSWC Report CARDIVNSWC-TR-94/004 (Mar 1994).

REVIEW OF IR SCALING LAWS

To begin a study of the infrared emissions from a naval ship by building a small-scale replicate of the ship requires that the model be identical to the full-scale ship, i.e., there must be geometrical similarity between the two. But this is not sufficient to the purpose considered because the "source" of the IR radiation is the temperature function T, which varies with position and time. In addition to geometrical similarity alone, it is desired therefore that the temperature behaviors be similar between full-scale ship and model, i.e., thermodynamic similarity is also a requirement. Clearly, the temperature variations across a ship are matters of some complication depending as they do upon the interaction of various conduction processes throughout the hull structure and of convective and radiative exchanges at the hull-fluid interfaces. A theory of scaling must therefore be developed that ensures thermodynamic similarity between a full-scale ship and its scaled model.

A key insight into developing such a theory involves recognition that the heat equation and its applicable boundary conditions are sufficiently general to contain a description of all the relevant processes of conduction, convection, and radiation at play in the determination of a ship's temperature function. These equations are expressed as:

$$\nabla \cdot [k(x) \nabla T(x,t)] + H(x,t) = \rho(x) c(x) \frac{\partial T}{\partial t}(x,t) \tag{1}$$

$$k(x) \nabla_n T(x,t) = h(x) \Delta T(x,t) \qquad \text{for all } x \subset b \tag{2}$$

$$T(x,t)\big|_{t=0} = T_o(x) \qquad \text{for all } x \tag{3}$$

where:

b	=	boundary
∇	=	three-dimensional gradient operator
∇_n	=	operator yielding gradient magnitude in direction normal to boundary
k	=	thermal conductivity (power/distance/temperature)
T	=	temperature at position x
x	=	coordinates of a point in three-dimensional position space
t	=	coordinates of a point in time
H	=	rate of heat production per unit volume at position x and time t (power/volume)
ρ	=	mass density at position x (mass/volume)
c	=	specific heat at position x (heat/temperature/mass)

h = heat transfer coefficient at position x, belonging to boundary b (power/area/temperature differential)

ΔT = temperature difference prevailing between ship and its surroundings, at position x and time t

T_o = initial temperature distribution that holds at every position x throughout the ship

The solution to the heat equation is a function of position and time and, furthermore, depends upon the heat sources, material thermodynamic properties, and ship-environment temperature differences, i.e.,

$$T = T(x,\ t,\ H,\ k,\ \rho,\ c,\ h,\ \Delta T) \tag{4}$$

The IR scaling laws are obtained by insisting that the heat equation and its boundary conditions shall hold in precisely the same form for both a full-scale ship and its scaled model. A connection between the equations, descriptive of full-scale ship and model, is obtained by use of scaling ratios that are assumed to hold for all variables. For example, the scaling ratio for the distance variables is

$$K_x = \frac{x^M}{x^S} \tag{5}$$

where K_x symbolizes a scaling ratio, the subscript x symbolizes the particular magnitudes of the distance vector, and the superscripts M or S refer to a model or ship variable, respectively. Not just x, but all the variables t, T, k, ρ, c, h, ΔT, and T_o are scaled according to ratios analogous to that of Equation (5) for the position vector magnitude. Scaling all the variables in this way allows a comparison of the heat equations and boundary conditions that describe the full-scale ship and its scaled model. If both sets of equations are to be strictly satisfied, their comparison shows that certain relationships must prevail among the various scale factors. These are called the IR scaling laws and have the form

$$K_{\frac{k}{\rho c}} \frac{K_t}{K_x^2} = 1 \tag{6}$$

$$\frac{K_H K_t}{K_{\rho c} K_T} = 1 \tag{7}$$

$$K_{\frac{h}{k}} K_x \frac{K_{\Delta T}}{K_T} = 1 \tag{8}$$

$$\frac{K_T}{K_{T_o}} = 1 \tag{9}$$

Adherence to these scaling laws is required for thermodynamic similarity to hold. When these conditions are met the temperature behavior of a ship and its scaled model are identical.

The information inherent in the scaling laws may be reformulated in a manner particularly suited to the description of an IR scaling experiment. The reformulation is based upon a transformation to dimensionless variables, the form of which is suggested by the scaling laws. The dimensionless variables, indicated by a bar above the variable symbols, are defined to be,

$$\bar{T} = \frac{T - T_\infty}{T_o - T_\infty} \tag{10}$$

$$\bar{x} = \frac{x}{L} \tag{11}$$

$$\bar{t} = \frac{k}{\rho c} \frac{t}{L^2} \qquad \text{(Fourier number)} \tag{12}$$

$$\bar{H} = \frac{H L^2}{k (T_o - T_\infty)} \tag{13}$$

$$\bar{h} = \frac{hL}{k} \qquad \text{(Biot number)} \tag{14}$$

These dimensionless variables are also called π variables in recognition of the role they play within Buckingham's π-theorem. It can be shown that the IR scaling laws hold provided the dimensionless variables have equal magnitude on both full-scale ship and scaled model. Thus, the entire process of designing IR scaled signature experiments involves arranging experimental circumstances so that such an equality of dimensionless variables may hold.

It is particularly satisfying that use of dimensionless variables transforms the heat equation and its boundary conditions to forms that are totally independent of the size of the ship to which they are applied, namely,

$$\bar{\nabla}^2 \bar{T} + \bar{H} = \frac{\partial \bar{T}}{\partial \bar{t}} \tag{15}$$

$$\bar{\nabla}_n \bar{T} = \bar{h} \bar{T} \quad \text{for all } \bar{x} \subset b \tag{16}$$

$$\bar{T}|_{\bar{t}=o} = 1 \quad \text{for all } \bar{x} \,. \tag{17}$$

Since in these last equations only the "barred" variables appear, they are said to be in manifestly dimensionless form. Clearly, they can apply in the same way to any ship whatever its size. They are therefore particularly useful in their applications to description of scaled model experiments. They have the same solution for ship or scaled model, which is

$$\bar{T} = \bar{T}(\bar{x}, \bar{t}, \bar{H}, \bar{h}) \,. \tag{18}$$

The dimensionless heat equation and its solution $\bar{T}$ are full information equivalents of the original heat equation and its solution, but are simply rendered in a form most suitable for application to scaling. The study of scaled IR phenomena may be advantageously based upon the determination of $\bar{T}$, because it does not explicitly depend upon size. The magnitude of $\bar{T}$ is completely fixed by the dimensionless variables $\bar{x}$, $\bar{t}$, $\bar{H}$, $\bar{h}$. To equate these between ship and scaled model is to similarly equate $\bar{T}$'s, and thus ensure perfect thermodynamic similarity.

Thus, the goal of IR scaling is reduced to the experimentally enforced equivalence between ship and model of these four dimensionless variables $\bar{x}$, $\bar{t}$, $\bar{H}$, and $\bar{h}$.

As a final point of this review, it is noted that $\bar{h}$, which "controls" the heat exchanges at the ship-fluid interface, has both a convective and a radiative component. This is also true of h, since

$$h = h_c + h_r \ . \tag{19}$$

In this equation,

$$h_c = \frac{k_f}{L_f} Nu(Gr,\ Pr,\ Re) \ . \tag{20}$$

The Nusselt number, Nu, is dimensionless, and in the most general case depends on three dimensionless numbers:

$$Re = \frac{\rho L_f v}{\mu} \qquad \text{(Reynolds number)} \tag{21}$$

$$Pr = \frac{\mu c}{k_f} \qquad \text{(Prandtl number)} \tag{22}$$

$$Gr = \frac{\beta g \ \Delta T \ L_f^3 \rho^2}{\mu^2} \qquad \text{(Grashof number)} \tag{23}$$

where:

ρ	=	fluid mass density
L_f	=	a representative length over which the fluid flows
v	=	fluid free stream velocity
μ	=	viscosity
c	=	fluid heat capacity
k_f	=	fluid thermal conductivity
ß	=	fluid thermal expansion coefficient
g	=	acceleration constant due to gravity
ΔT	=	temperature differential between fluid and surface

Note in Equation (19) that h_r satisfies

$$h_r = \frac{4k}{L} St$$

(24)

and thus depends upon the dimensionless Stefan number

$$St = \frac{L}{k}\epsilon\sigma T'^3 \tag{25}$$

where:

ϵ	=	emissivity of surface
σ	=	Stefan constant
T'	=	average temperature of an emitting surface and its background

In summary, both convective and radiative contributions can be incorporated into the Biot number in accordance with the expression

$$\bar{h} = \frac{L}{k}\left[\frac{k_f}{L_f}Nu + \frac{4k}{L}St\right] . \tag{26}$$

AN ERROR OF SCALE

In the theory of scale modeling, a number of dimensionless variables occur that apply to both a full-scale object and its scaled model. The physical behavior of the scaled model is faithful to that of the prototype on condition that all of the dimensionless variables are identical for the full-scale object and the scaled model. In practice it is not uncommon that difficulties occur in trying to maintain equality of all the dimensionless variables of the full-scale object and the scaled model. In such a case, the equality requirement for one or more of the dimensionless variables may have to be relaxed. Such a relaxation is referred to as an "error" of scale because the strict scaling requirements are not followed exactly. Common experience with scaling experiments indicates that scaling errors need not invalidate results obtained from such experiments. In such cases, however, it is important to be aware of the presence of such scaling errors and to interpret correctly their impact upon measurement results.

In deriving the dimensionless variables that control the scale model experiments concerned with IR radiation, it is assumed that lengths in all directions are scaled in the same way. But in production of a scaled model, this can in some cases be a severe requirement that is difficult or expensive to strictly observe. A typical naval ship is hundreds of feet long, and the hull walls are made of metal plate that is less than one inch thick. Then, in scaling the ship to a convenient laboratory size (a few feet), it is easy to see that scaling the hull thickness and length similarly, as is required by strict adherence to theory, leads to a very thin hull thickness (about the thickness of a sheet of writing paper). Thus a convenient laboratory length may raise conflicts associated with the very thin hull required by exact scaling theory.

The difficulty encountered can be illustrated by a numerical example. Consider scaling a patrol gunboat of length 160 ft having a plate aluminum hull thickness of 0.75

in. Adopting a common scale ratio equal to 1/48 yields for a model a length of 3.33 ft and a hull thickness of 0.016 in. While the model length may be quite convenient, the model hull would be so thin as to present problems of structural strength. For example, if fiberglass were to be used as a modeling material, it would be both difficult and expensive, using standard methods and materials, to produce such a thin hull. The result would be a hull the strength of which might not be adequate to the task of withstanding stressful laboratory experiments, such as moving through water at high velocity.

For these reasons, it appears that scaling both hull thickness and length in the same way, as demanded by the exact IR scaling requirements, can in some cases cause problems. Thus, it must be determined if it is possible to scale hull thickness and length differently. An interpretation must be found under which it is possible to meaningfully carry out experiments with different scales of length in directions orthogonal and parallel to the hull surface.

In general, it will not be physically meaningful to scale differently in different directions. This is because the exact IR scaling laws, as they have been derived, require a common scale applied to the three independent directions of space. In one approximation, however, this requirement may be relaxed. Suppose there exists a temperature gradient that is less, by several orders of magnitude, in directions parallel to hull surfaces than in directions orthogonal to such surfaces. It is then reasonable to neglect the flow of heat parallel to the hull, and to consider only that flow of heat perpendicular to it. In such a case, the gradient of the temperature, as it occurs in the heat equation and its companion boundary condition, is reduced to the simpler gradient normal to the hull surface. If the derivation of the IR scaling laws is followed through as originally derived, dimensionless variables are obtained of the exact form as previously described, but now with quite a different interpretation. The dimensionless variables are, analogously to Equations (10) to (14),

$$\bar{T}=\frac{T-T_\infty}{T_o-T_\infty} \tag{27}$$

$$\bar{x}=\frac{x}{L} \tag{28}$$

$$\bar{t}=\frac{k}{\rho c}\frac{t}{L^2} \tag{29}$$

$$\bar{H}=\frac{HL^2}{k(T_o-T_\infty)} \tag{30}$$

$$\bar{h} = \frac{hL}{k} \tag{31}$$

All symbols retain their previous definitions, with the exception of the symbols x and L. Previously, L symbolized a typical length along any arbitrary direction. Now the meaning of L is restricted to be a typical length directed orthogonally to a hull surface. Previously, x symbolized a position vector along any arbitrary direction. Now the meaning of x is such that it is restricted to lie along a direction orthogonal to a hull surface.

Of these dimensionless variables, only $\bar{h}$ is affected by the scale of length that applies to directions parallel to a hull surface. Such dependence arises through the term describing convection. If $\bar{h}$ is written as a sum of convective and radiative effects, respectively, then similarly as in Eq. (26),

$$\bar{h} = \frac{L}{k}\left[\frac{k_f}{L_f}Nu + \frac{4k}{L}S_t\right] . \tag{32}$$

In this expression, all symbols retain their previous definitions, except that L_f now symbolizes a typical length over which fluid flows and is explicitly assumed to lie in a direction parallel to a hull surface. Note that L and L_f, in the approximation being considered, will not represent the same scale of lengths as had been the case.

Now the Nusselt number, Nu, will depend upon L_f. Recall that

$$Nu = Nu(Re,\ Pr,\ Gr) \tag{33}$$

where, analogously to Equations (21) to (23),

$$Re = \frac{\rho L_f v}{\mu} \tag{34}$$

$$Pr = \frac{\mu c}{k_f} \tag{35}$$

$$Gr = \frac{\beta g\ \Delta T\ L_f^3 \rho^2}{\mu^2} \tag{36}$$

Again, in these last expressions all symbols retain their previous definitions, but L_f takes on the new and restricted meaning just given of defining a scale of length parallel to a hull surface. Thus, Re and Gr, and through them Nu, are affected by the scale of distance parallel to a hull surface.

As is always the case with scale modeling, the dimensionless variables must be set equal between the full-scale object and corresponding model so that, for example,

$$\bar{h}^M = \bar{h}^S \tag{37}$$

is a requirement for faithful scale model behavior. In contrast to the exact modeling case, however, within the approximation considered, not one scale of distance but rather two scales of distance are involved in the enforcement of the last equality. These scales are represented by L and L_f typical distances restricted, respectively, to lie along directions orthogonal to and parallel to a hull surface.

Within the approximation considered, namely that all heat conduction is orthogonal to a hull surface, it is acceptable to have two different scales of length enter the scaling formalism in the manner described. The basic physical interpretation that arises is one in which the two different length scales apply to two different heat transfer processes, specifically, conduction and convection. The scaling of the conduction process is fixed by L. For a given L, the scaling of convection is fixed by L_f, since Re and Gr are dependent upon L_f, and Nu is in turn a function of this pair of dimensionless variables. If heat conduction occurs only orthogonally to a hull surface, it is suitable that only the scale of distance in that direction affects the scaling of the conduction process. On the other hand, a variable like Re, which controls the pattern of flow about the hull and thus affects convection, is not affected by the thickness of the hull but rather by the dimensions that lie in the surface itself and are in physical contact with the fluid flow. Thus Re is a function of L_f. Therefore for fixed L, the scaling of convective processes is controlled by the scale of distance parallel to the hull surface.

As a precautionary note, the above approximation and interpretation would apply only on condition that the large temperature gradients lie in directions orthogonal to hull surfaces. It is not unreasonable, however, to expect that for many important ship operating conditions such circumstances would hold. In fact, this approximation generally is used in most infrared ship signature computer programs.

The value in applying different length scales to conductive and convective processes lies in the practical use of such concepts. This application offers the possibility of reduced time and costs associated with a scale modeling experiment that would satisfy the exact IR scaling laws. In all circumstances, practical considerations aside, it is preferred that the exact scaling laws be strictly enforced. But in those circumstances where the practical circumstances alluded to above are determining factors, a certain number of scaling experiments may still be undertaken within the approximations discussed.

SURVEY OF THERMODYNAMIC PROPERTIES RELEVANT TO IR SCALING

The successful use of previously derived IR scaling laws for the design of IR experiments requires a knowledge of the thermal properties of materials used in construction of full-scale ships and their scaled models. This arises because scaled models in such experiments are generally not made of the same materials as their full-scale ship equivalents. In order to have the scaled model behave thermodynamically in a way that perfectly mimics the full-scale ship, within convenient laboratory magnitudes of space, time, energy production and dissipation, it is necessary to select the thermal properties of the model suitably. Which thermal properties are the relevant variables and what their magnitudes should be for a given IR scaling experiment are determined by the dimensionless variables that underlie such experiments. The relevant forms of the dimensionless variables are reviewed here to emphasize the thermal properties indicated by square brackets:

$$\bar{T} = \frac{T - T_\infty}{T_o - T_\infty} \tag{38}$$

$$\bar{x} = \frac{x}{L} \tag{39}$$

$$\bar{t} = \frac{[k]}{[\rho][c]} \frac{t}{L^2} \tag{40}$$

$$\bar{H} = \frac{HL^2}{[k](T_o - T_\infty)} \tag{41}$$

$$\bar{h} = \frac{[h]L}{[k]} \tag{42}$$

The symbols used retain their previously defined meanings. The dimensionless temperature $\bar{T}$ is a function of the other dimensionless variables $\bar{x}$, $\bar{t}$, $\bar{H}$, $\bar{h}$. A goal of a scaled IR experiment is to make the dimensionless temperature function $\bar{T}$ the same for full-scale ship and scaled model, which is tantamount to their regular temperatures T being functionally the same, as long as the experiment is carried out under equivalent initial and boundary conditions as represented by T_o and T_∞, respectively. Of course, the temperature, in accordance with the Planck black-body radiation law, is the source of the IR radiation. Hence, if the temperature functions T are equivalent for full-scale ship

prototype and its scaled model, the IR contrast of the ship may be determined by studying that of the scaled model instead. These temperature functions will be equal, as is known, if the dimensionless variables on which they depend are made equal, i.e., if $\overline{x}$, $\overline{t}$, $\overline{H}$, and $\overline{h}$ are made equal. By examining the definitions of these variables, it is found that for given experimental values of position, time, and heat energy the thermodynamic properties that fix the magnitude of the dimensionless variables are k, ρ, c, and h.

These thermodynamic properties are to be selected in the design of an IR scaled model as they control the equality to be imposed on the dimensionless variables of ship and scaled model. Also note, as in Equation (19), that the last of these, the heat transfer coefficient, h, may be broken into its convective and radiative components, i.e.,

$$h = h_c + h_r \tag{43}$$

Recall from Equations (24) and (25) that the radiation transfer coefficient, at a given temperature, is controlled by the surface emissivity, ϵ. The relative range of h_r is therefore determined by the corresponding range of ϵ. It is convenient to consider convective and radiative effects separately. Thus interest would be in the range of values that may commonly be adopted in scale model IR experiments by the variables k, ρ, c, h_c, and ϵ.

Next, the range of values taken by these variables are tabulated using ordinary materials and conditions for IR scale model experimentation. Such data are widely available in common thermodynamic textbooks and reference collections. The reason for collecting the material here is to select useful material from many sources, the whole of which would be inconvenient to obtain, and to render it into one consistent set of units. In this way the properties are set in a convenient form for the designer of IR scaled experiments. English units are used throughout, although a table of conversion factors is provided to allow conversions to other units if desired.

In the tables, standard conditions of room temperature and pressure of one atmosphere are assumed. The property tables are intended only to convey the correct order of magnitudes and the correct qualitative comparison of magnitudes among the various materials and conditions considered.

The properties k, ρ, c_p are displayed in Tables 1, 2, and 3 for metals, alloys, and non-metals, respectively. The thermal diffusivity, α, which is the algebraic combination of k, ρ, c that enters the Fourier number, is also listed in these tables.

In Table 4, the range of values to be expected for the convective heat transfer coefficient is displayed. The fluids of principal interest in scaled laboratory experiments will likely be air and water, and it is the range of h_c for these which is listed. Additional properties of general interest for air and water are listed in Tables 5 and 6, respectively. Note, however, the possibility of using other fluids in scaled IR experiments.

As stated, the radiation contribution to the heat transfer coefficient, h_r, is determined for given temperatures by the surface emissivity, ϵ. Values of ϵ are listed in

Tables 7 and 8, for various metal and non-metal surfaces, respectively. Useful conversion factors are listed in Table 9.

SUMMARY AND CONCLUSIONS

An error of scale and the tabulation of certain thermodynamic properties are presented. In order to establish the context in which these are matters of interest, the principal ideas on which the practice of IR scaling experiments may be based are reviewed. The role of dimensionless variables in the design, control, and interpretation of such experiments is emphasized. The relevant dimensionless variables are defined in Equations (10) to (14). The experimentally enforced equality on full-scale ship and model of these dimensionless variables ensures thermal similitude, and hence forms the basis of the study of IR contrast by means of scaled models.

In the derivation of the scaling laws and the dimensionless variables which flow from them, it is assumed that all spatial variables are scaled in the same way. However, a practical problem may occur when a "long" ship with a "thin hull" is isotropically scaled. A model of convenient laboratory length may correspond to a hull which is only a few mils thick, and thus not withstand stressful laboratory experiments. Although isotropic scaling, in accordance with the exact derivation of the IR scaling laws, is much to be preferred, one circumstance occurs in which a suitable approximation allows different scales of length to be employed orthogonal and parallel to the hull surface. The circumstance occurs if the only substantial temperature gradients are orthogonal to the hull surfaces. The essential result which follows is that conductive effects and convective effects are "controlled" separately by different scales of length. The dimensionless variables of IR scaling are reinterpreted in light of the approximation incorporating two length scales. In the isotropic case the typical lengths L and L_f, which enter the dimensionless variables, have the same meaning and set the same scale of length. In the approximation presented, the two typical lengths L and L_f are no longer equal and fix different scales of length orthogonal and parallel, respectively, to the hull surfaces. It is expected that the physical interpretation given which, as an approximation, assigns separate scales of length to conductive and convective effects will find some use in applications where circumstances weigh in favor of a quick and rough study of IR before a much larger commitment of resources would be indicated.

Finally, a range of values for thermal properties using ordinary materials and conditions for IR scaled model experiments are presented. The determination of which thermal properties would be useful was guided by the form of the dimensionless variables. Upon examination it is found that, for given conditions of heat load, position, and time, the magnitudes of the dimensionless variables depend upon the properties k, ρ, c, h_c, and ϵ. Hence, these properties are tabulated for a variety of materials and conditions. The properties have been collected from standard references and are presented in a uniform set of units. Although no claim is made for any great accuracy in these tabulations, their usefulness would be based upon correct orders of magnitude and a correct qualitative ordering of property magnitudes among the various materials and conditions considered.

Table 1. Metals - Thermal properties k,ρ,c,α.
(thermal conductivity k, mass density ρ, heat capacity c, and thermal diffusivity α)

Metals	k, Btu/hr·ft·°F	ρ, lb/ft^3	c, Btu/lb·°F	α, ft^2/hr
Aluminum	117.0	169	0.208	3.33
Antinomy	10.6	415	0.049	0.52
Bismuth	4.9	612	0.029	0.28
Cadmium	54.0	540	0.055	1.82
Copper	224.0	558	0.091	4.42
Gold	169.0	1203	0.030	4.68
Iron	35.8	491	0.104	0.70
Inconel	8.7	534	0.109	0.15
Lead	20.1	705	0.030	0.95
Magnesium	91.0	109	0.232	3.60
Mercury	4.8	849	0.033	0.17
Molybdenum	72.0	638	0.060	1.88
Nickel	52.0	556	0.106	0.87
Palladium	40.6	743	0.054	1.01
Platinum	40.2	1340	0.032	0.94
Silver	241.0	655	0.056	6.57
Tin	38.0	456	0.054	1.54
Tungsten	94.0	1208	0.032	2.43
Zinc	65.1	446	0.091	1.60

Table 2. Alloys - Thermal properties k,ρ,c,α.
(thermal conductivity k, mass density ρ, heat capacity c, and thermal diffusivity α; units are listed in Table 1.)

Alloys	k	ρ	c	α
Aluminum				
Al-Cu, 94-6	95.0	174	0.211	2.587
Al-Si-Cu, 87-12-1	79.0	166	0.207	2.299
Al-Si, 80-20	93.0	164	0.204	2.779
Al-Mg-Si-Mn, 97-1-1-1	102.0	169	0.213	2.833
Copper				
Brass, Cu-Zn, 70-30	61.5	532	0.092	1.260
Red brass, Cu-Sn-Zn, 85-9-6	35.0	544	0.092	0.699
Bronze, Cu-Sn, 75-25	15.0	541	0.082	0.340
Aluminum bronze, Cu-Al, 95-5	48.0	541	0.098	0.903
Constantan, Cu-Ni, 60-40	12.4	557	0.100	0.220
German silver, Cu-Ni-Zn, 63-15-22	14.0	538	0.094	0.284
Iron				
Cast iron	33.0	474	0.110	0.630
Wrought iron	34.0	490	0.110	0.630
Steel				
18-8 Stainless type 304	8.0	488	0.110	0.150
Carbon steel, C, 0.5	31.0	489	0.111	0.571
C, 1.0	25.0	487	0.113	0.454
C, 1.5	21.0	484	0.116	0.376
Nickel steel, Ni, 0	42.0	493	0.110	0.785
Ni, 20	11.0	495	0.110	0.204
Ni, 36	6.2	508	0.110	0.111
Ni, 40	5.8	510	0.110	0.108
Ni, 80	20.0	538	0.110	0.338

Table 2. (Continued)

Alloys	k	ρ	c	α
Chrome steel, Cr, 0	42.0	493	0.110	0.780
Cr, 1	35.0	491	0.110	0.640
Cr, 5	23.0	489	0.110	0.430
Cr, 20	13.0	480	0.110	0.240
Chrome-nickel steel, Cr-Ni, 15-10	11.0	491	0.110	0.200
Cr-Ni, 18-8	9.4	488	0.110	0.170
Cr-Ni, 20-15	8.7	489	0.110	0.160
Cr-Ni, 25-20	7.4	491	0.110	0.140
Tungsten steel, W, 0	42.0	493	0.110	0.780
W, 1	38.0	494	0.110	0.720
W, 5	31.0	504	0.100	0.590
W, 10	28.0	519	0.100	0.530
Nickel-Chromium				
Ni-Cr, 90-10	10.0	541	0.106	0.170
Ni-Cr, 80-20	7.3	519	0.106	0.130

Table 3. Non-Metals - Thermal properties k,ρ,c,α.
(thermal conductivity k, mass density ρ, heat capacity c, and thermal diffusivity α)

Non-Metals	k, Btu/hr·ft·°F	ρ, lb/ft^3	c, Btu/lb·°F	α, ft^2/hr
Aerogel Silica	0.012	8.5	0.200	0.0070
Asbestos	0.087	36.0	0.250	0.0100
Bakelite	0.134	79.5	0.380	0.0040
Clay	0.739	91.0	0.210	0.0390
Coal, anthracite	0.150	85.0	0.300	0.0060
powdered	0.067	46.0	0.310	0.0050
Cork	0.025	10.0	0.040	0.0060
Cotton	0.034	5.0	0.310	0.0750
Glass				
Ordinary	0.510	162.0	0.160	0.0200
Pyrex	0.650	140.0	0.200	0.2300
Fuzed quartz	0.820	137.0	0.160	0.0360
Glass wool	0.021	1.5	0.200	0.0700
Ice	1.280	57.0	0.460	0.0480
Plasterboard	0.250	78.1	0.200	0.0160
Rubber, hard	0.087	75.0	0.339	0.0034
Soil				
Calcerous earth, 43% H_2O	0.410	104.0	0.530	0.0070
Quartz sand, medium fine, dry	0.150	103.0	0.190	0.0080
Quartz sand, 83% H_2O	0.340	109.0	0.240	0.0130
Sandy clay, 15% H_2O	0.530	111.0	0.330	0.0150
Mud, wet	0.500	94.0	0.600	0.0080

Table 3. (Continued)

Non-Metals	k, Btu/hr·ft·°F	ρ, lb/ft^3	c, Btu/lb·°F	α, ft^2/hr
Wood, cross grain				
Ash	0.102	40.0	0.430	0.0057
Birch	0.099	44.0	0.380	0.0059
Hemlock	0.080	34.0	0.490	0.0048
Oak	0.120	51.0	0.570	0.0041
Pine	0.060	31.0	0.670	0.0029
Spruce	0.070	26.0	0.300	0.0090

Table 4. Typical range of values for convective heat transfer coefficient h_c (Btu/hr·ft^2·°F).

Fluid	Free Convection	Forced Convection
Air	3-17	17-170
Water	17-170	170-5700
Condensing Water Vapor	1,700-17,000	1,700-114,000

Table 5. Properties of water at 20°C.

k, Btu/hr·ft·°F	ρ, lb/ft^3	c, Btu/lb·°F	α, ft^2/hr	ν, ft^2/sec	Pr	ß, 1/°R
0.345	62.46	0.9988	5.54×10^{-3}	1.083×10^{-5}	7.02	1.0×10^{-4}

Table 6. Properties of air at 20°C.

k, Btu/hr·ft·°F	c, Btu/lb·°F	μ, lb/hr·ft	Pr
0.0143	0.240	0.0427	0.712

Table 7. Metals - Surface emissivity.

Metals		ϵ
Aluminum	polished	0.039
	oxidized	0.200
Brass	polished	0.030
	oxidized	0.610
Chromium	polished	0.075
Copper	polished	0.023
	oxidized	0.780
Gold	polished	0.020
Iron	polished	0.210
	oxidized	0.740
	tin-plated	0.070
Lead	polished	0.060
	oxidized	0.630
Nickel	polished	0.072
	oxidized	0.350
Steel	polished stainless	0.074
	oxidized	0.790
Titanium		0.100
Tungsten		0.110

Table 8. Non-Metals - Surface emissivity.

Non-Metals	ϵ
Asbestos	0.950
Brick, rough, red	0.930
Clay	0.910
Coal, soot	0.950
Concrete	0.940
Concrete Tiles	0.630
Glass, smooth	0.940
Ice, smooth	0.950
Limestone	0.900
Marble	0.940
Paints, Lacquers, Varnishes	
Aluminum	0.550
Oil (all colors)	0.900
Black lacquer on iron	0.880
Black shellac on tinned iron	0.820
Black matte shellac	0.910
Paper	0.890
Plasterboard	0.920
Porcelain, glazed	0.920
Quartz, rough, fused	0.930
Roofing Paper	0.910
Rubber	0.930
Sandstone	0.870
Wood, oak	0.900

Table 9. Conversion factors.

Length	
1 cm	0.3937 in.
1 m	3.2808 ft
Area	
1 sq cm	0.1550 sq in.
1 sq m	10.7639 sq ft
Volume	
1 cu cm	0.0610 cu in.
1 cu m	35.3145 cu ft
Force	
1 dyne x 10^6	1.020 kg
1 kg	2.205 lb
Mass	
1 slug	32.17 lb
1 kg	0.06854 slug
Energy	
1 erg or dyne-cm	10^{-7} watt-sec
1 joule	1 watt-sec
1 g cal	4.186 watt-sec
1 ft-lb	1.356 watt-sec
1 Btu	778.3 ft-lb
1 Btu	1054.8 watt-sec
1 watt	3.413 Btu/hr
1 kcal	3.968 Btu
Heat capacity	
1 joule/(g)(°C)	0.23895 Btu/(lb)(°F)
1 g cal/(g)(°C)	1 Btu/(lb)(°F)
1 Btu/(lb)(°F)	1900 watt-sec
Thermal conductivity	
1 cal/(sec)(cm)(°C)	241.9 Btu/(hr)(sq ft)(°F/ft)
1 kcal/(hr)(m)(°C)	0.672 Btu/(hr)(sq ft)(°F/ft)
1 watt/(cm)(°C)	57.79 Btu/(hr)(sq ft)(°F/ft)

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